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Geodesics for Photons, Particles, and Tachyons in a Cosmological Model

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We find geodesics in a flat universe obeying a perfect gas law for the equation of state, by means of a constant deceleration model. Inflationary and power-law cases are considered.

1. INTRODUCTION

The most recently study of geodesics in the isotropic universe was undertaken by Chaliassos (1987), who studied the analytic form of the geodesics in the Robertson–Walker metric, and also found the solutions for the closed, flat, and open Friedmann models, in the dust and radiation phases, with a null cosmological constant. In this note we consider geodesics in the inflationary phase, along with the solution for the general case of a perfect gas equation of state,

$$p = \gamma\rho \qquad (\gamma = \text{const}) \tag{1}$$

where p and ρ stand for cosmic pressure and rest-energy density, respectively.

Such a law occurs in cosmological flat models when one selects constant deceleration parameter models,

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = m - 1 = \text{const} \tag{2}$$

where $R(t)$ is the scale factor in the Robertson–Walker metric,

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{3}$$

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In Section 2 we study null geodesics, in Section 3, particles and tachyons.

2. NULL GEODESICS

The general equation for null geodesics is (Chaliassos, 1987)

$$r = \pm \int \frac{dt}{R} \tag{4}$$

For the inflationary phase ($m = 0$),

$$R = R_0 e^{Ht} \qquad (R_0,\ H \text{ constants}) \tag{5a}$$

$$\gamma = -1 \tag{5b}$$

and

$$r = \pm \frac{e^{-Ht}}{R_0 H} + r_0 \qquad (r_0 = \text{const}) \tag{6}$$

We interpret this result as meaning that a photon in the inflationary phase will be approximately comoving, and

$$r = r_0 \tag{7}$$

In other words, photons are "localized."

Let us now consider the $m \neq 0$ cases, where a power law applies for $R(t)$ (Berman, 1983; Berman and Gomide, 1988)

$$R(t) = (mDt)^{1/m} \tag{8a}$$

$$\gamma = \tfrac{1}{3}(2m - 3) \tag{8b}$$

From (4), we find, for null geodesics, if $m \neq 1$,

$$r = \pm \left[\left(1 - \frac{1}{m}\right)(mD)^{1/m} \right]^{-1} t^{1-1/m} + r_0 \tag{9}$$

This is a monotonic increasing function of t for $m > 1$. When $t = 0$, $r = r_0$; for $t \to \infty$, we have $r \to \infty$, as expected. For $m < 1$, we have a photon at infinity when $t = 0$, which at $t \to \infty$ gives $r \to r_0$ (comoving photon).

For the case $m = 1$, we find

$$r = \pm (D)^{-1/2} \ln t + r_0 \tag{10}$$

In order to have a meaningful result, in this case we must choose the plus sign, and then, when $t \to \infty$, $r \to \infty$. We have to adjust r_0 so that for $t \to 10^{-43}$ sec, $r \to 0$ (comoving). For $t < 10^{-43}$ sec, our model does not apply, because we would be out of the classical domain.

3. GEODESICS FOR PARTICLES AND TACHYONS IN A FLAT UNIVERSE

Our solution considers the case where particles and tachyons live in a $p = \alpha\rho$ universe (α = const). This happens automatically if we adopt the hypothesis stated in Section 2 for flat universes with

$$\alpha = \frac{2m - 3}{3} \tag{11}$$

When $m = 3/2$ we recover the dust solution, and for $m = 2$ we get the radiation case, in flat universes. We can see that we are solving for a general perfect gas equation of state, because, by varying m, we vary α in (6).

Let us first look at the inflationary case, given by $m = 0$. The geodesic is given by

$$r = \pm \int \frac{dt}{R_0 e^{Dt}\{1 + \lambda\alpha'^2 R_0 e^{2Dt}\}^{1/2}} \tag{12}$$

where $\alpha' = m'^2c^2/p_r^2$ = const. Here m' is the rest-mass and p_r the radial momentum. When we plug into (8) the reasonably widely accepted numerical data for the D, t range, and suppose R_0 to be of order 10^{-33} cm (Planck's length), we verify that the integral reduces to the corresponding null geodesic case:

$$r \cong \pm \int R_0^{-1} e^{-Dt}\, dt \cong \text{const} \qquad (\text{"localized", or comoving particle.}) \tag{13}$$

Let us now consider the $m \neq 0$ cases. We have

$$r = \pm \int \frac{dt}{(mDt)^{1/m}\{1 + \lambda\alpha'^2(mDt)^{2/m}\}^{1/2}} \tag{14}$$

Two cases are amendable to analytic solution. When $m = 1$ and $\lambda = 1$ (particle),

$$r = \mp\frac{1}{D}\ln\left[\frac{\sqrt{t^2 + 1/K} + K^{-1/2}}{t}\right] \tag{15}$$

where $K = \alpha'^2 D^2$.

When $m = 1$ and $\lambda = -1$ (tachyons),

$$r = \mp\frac{1}{D}\ln\left[\frac{\sqrt{1/K - t^2} + K^{-1/2}}{t}\right] \tag{16}$$

In this case, it is evident that t has a maximum possible value, $t_{max} = K^{-1/2}$.

Two other cases ($m = 2$) have parametric solutions that were given by Chaliassos in analytic form: For $\lambda = 1$, $m = 2$,

$$r = \pm\frac{1}{\alpha'}\{\alpha' y + \sqrt{1 + \alpha'^2 y}\} \pm \text{const} \tag{17}$$

where

$$dy = R^{-1}\,dt \tag{18}$$

For $\lambda = -1$, $m = 2$,

$$r = \pm\frac{1}{\alpha'}\arcsin(\alpha' y) \tag{19}$$

It is expected that other cases, when treated numerically, yield similar results.

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